Spring Block 5

Area, perimeter and volume

White Rose MATHS

Small steps

Step 1	Shapes – same area
Step 2	Area and perimeter
Step 3	Area of a triangle – counting squares
Step 4	Area of a right-angled triangle
Step 5	Area of any triangle
Step 6	Area of a parallelogram
Step 7	Volume – counting cubes
Step 8	Volume of a cuboid

Shapes – same area

Notes and guidance

In this small step, children recap learning from previous years by finding the areas of shapes. It may be useful to remind children about the differences between area and perimeter, which will be covered explicitly in the next step.

Children find the areas of shapes by counting squares and then identify shapes that have the same area. It should become clear to children that shapes can look different but still have the same area. Rectilinear shapes are included here.

Children then explore instances when multiplication can be used to find the areas of shapes. They should begin to identify rectangles that will have the same area by using factor pairs rather than relying on counting squares. They can also use factor pairs to draw rectangles that have the same area.

Things to look out for

- Children may confuse area and perimeter.
- When counting squares, children may miscount or use inefficient strategies.
- Children may not use factor pairs to notice shapes that have the same area or to create shapes with the same area.

Key questions

- How can you find the area of this shape? Is there more than one way?
- Do shapes that have the same area have to look the same?
- How can you use factor pairs to find shapes that would have the same area?
- How would you draw more than one rectangle that has an area of _____ cm^2?

Possible sentence stems

- The total number of squares in the rectangle is _____

  The area of the rectangle is _____ cm^2
- The length of the rectangle is _____ cm.

  The width of the rectangle is _____ cm.

  The area of the rectangle is _____ cm^2

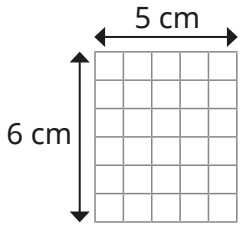
National Curriculum links

- Recognise that shapes with the same areas can have different perimeters and vice versa

Shapes – same area

Key learning

- Complete the sentences to describe the rectangle.



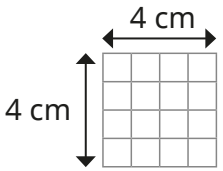
The length of the rectangle is _____ cm.

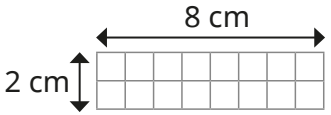
The width of the rectangle is _____ cm.

The total number of squares in the rectangle is _____

The area of the rectangle is _____ cm^2

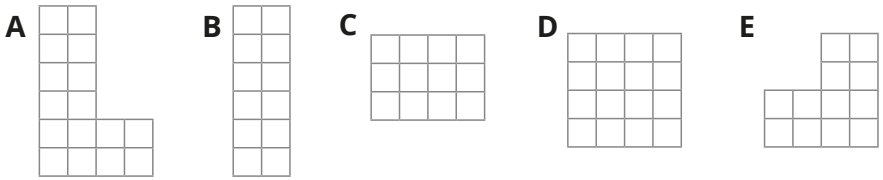
Use the same method to find the areas of these rectangles.





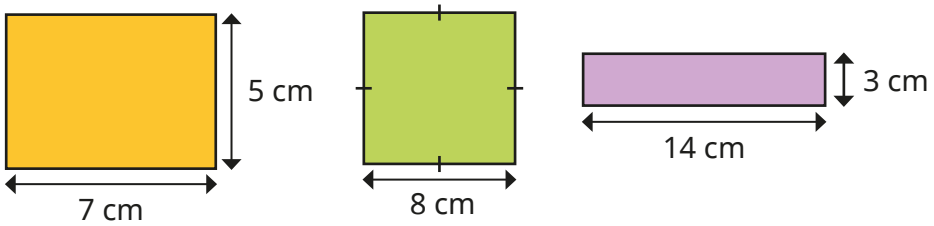
What do you notice?

- Each square represents 1 cm^2



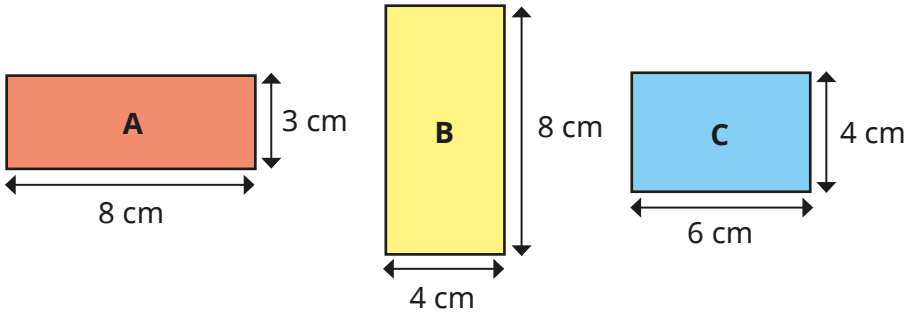
  - Which shapes have an area of 12 cm^2?
  - Which shapes have an area of 16 cm^2?
  - Why is there more than one representation for each?

- Find the areas of the rectangles.



Explain your method to a partner.

- Which two rectangles have the same area?



How do you know?

- Draw as many rectangles as possible that have these areas.

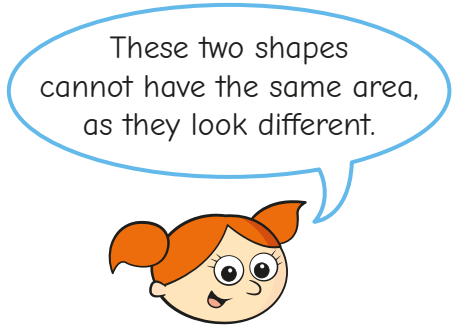
All the side lengths should be whole numbers.

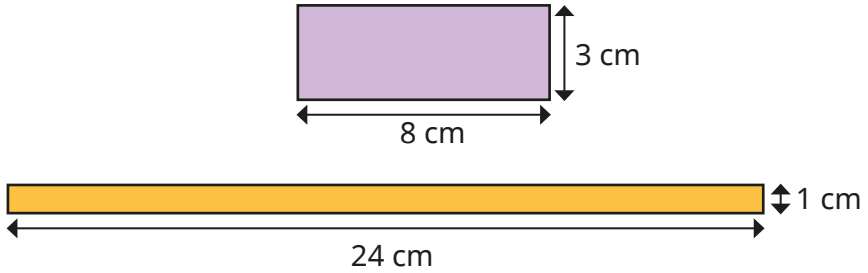
  - 36 cm^2
  - 16 cm^2
  - 17 cm^2

What do you notice about your last answer?

Shapes – same area

Reasoning and problem solving



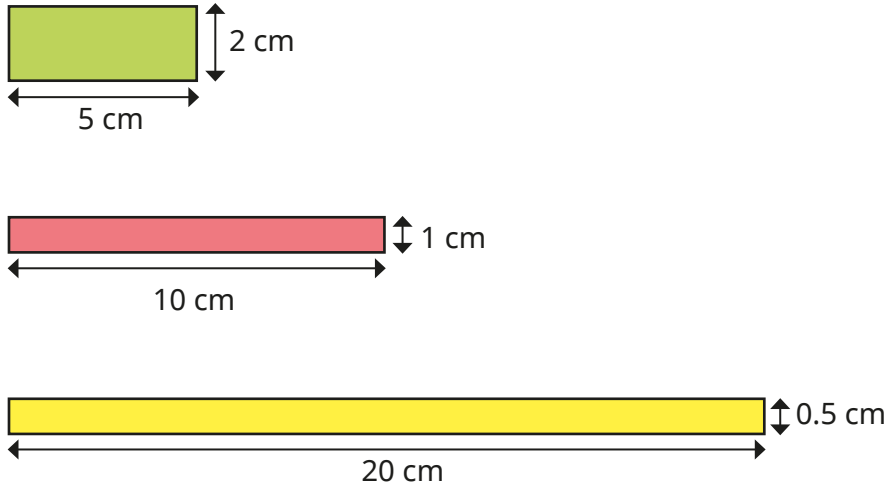


Do you agree with Alex?

Explain your answer.

No

Which rectangle has the greatest area?



Sketch the next rectangle in the pattern.

What is its area?

How do you know?

All the rectangles have the same area. | $10\ cm^2$

Area and perimeter

Notes and guidance

Building on the previous step and reinforcing learning from Year 5, in this small step children find the areas and perimeters of rectangles and rectilinear shapes.

Children explore methods for finding the perimeters and areas of rectangles and rectilinear shapes and compare their efficiency. When finding the area of a rectilinear shape, encourage children to look for the most efficient way to split the shape rather than always splitting it the same way. They should pay close attention when calculating unknown side lengths, and explain how they know whether they need to add or subtract. They can also explore when it may be efficient to find the area of a rectilinear shape by subtracting the missing part from the area of a whole rectangle.

Things to look out for

- Children may confuse area and perimeter.
- When finding the area of a rectilinear shape, children may not split the shape in the most efficient way.
- When calculating the perimeter, children may not use efficient strategies, instead relying on adding lengths in order.
- Children may struggle to work out missing side lengths or forget to do so.

Key questions

- What is perimeter? What is area?
- How can you find the perimeter of the rectangle?
- How can you find the area of the rectangle?
- What is the formula to find the area of a rectangle?
- How can you split the rectilinear shape into rectangles? Is there more than one way?
- How is finding the area/perimeter of a rectilinear shape different to finding the area/perimeter of a rectangle? How is it similar?
- How can you work out the other side lengths?

Possible sentence stems

- The formula to find the area of a rectangle is ...
- To find the perimeter of a rectangle, I ...

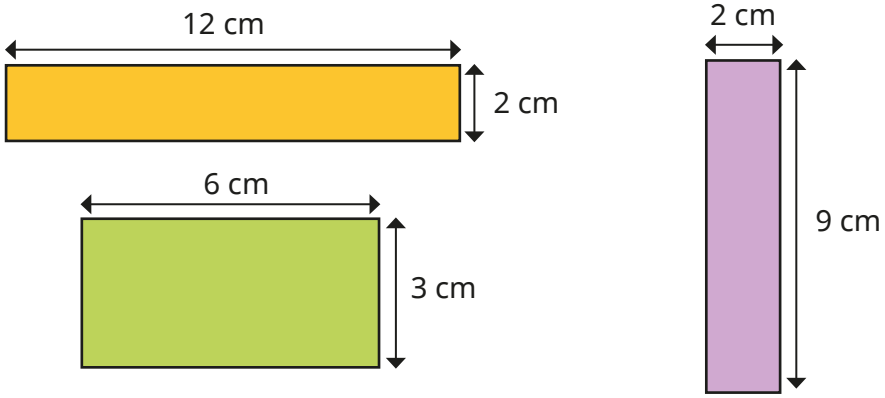
National Curriculum links

- Recognise that shapes with the same areas can have different perimeters and vice versa
- Recognise when it is possible to use formulae for area and volume of shapes

Area and perimeter

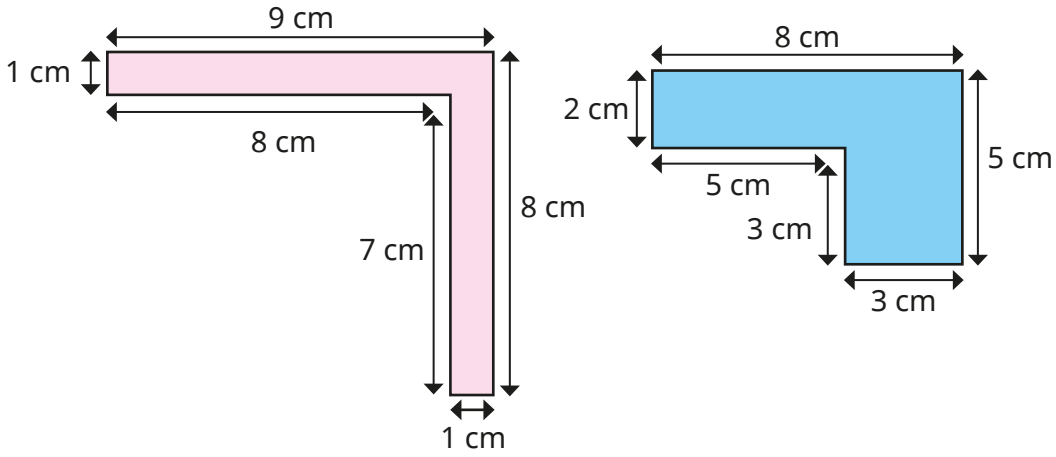
Key learning

- Find the area and perimeter of each rectangle.

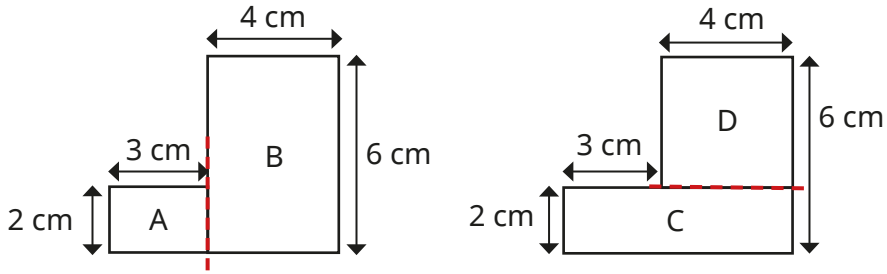


Compare methods with a partner.

- Work out the perimeters of the rectilinear shapes.



- Both of these rectilinear shapes are made from two rectangles.

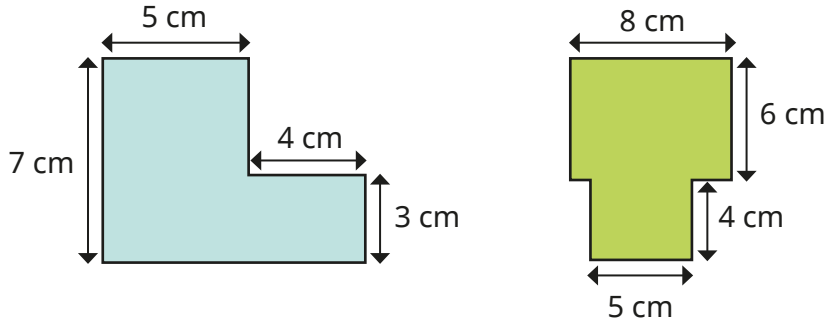


Work out the areas of the rectangles to work out the areas of the rectilinear shapes.

What do you notice?

Why does this happen?

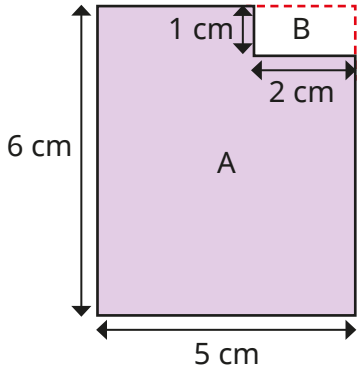
- Find the area and perimeter of each shape.



Area and perimeter

Reasoning and problem solving

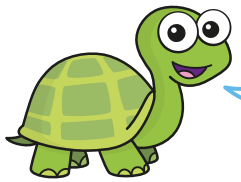
Tiny is finding the area of this shape.



Area of A = 6 cm × 5 cm
= 30 cm²

Area of B = 1 cm × 2 cm
= 2 cm²

Total area = 32 cm²



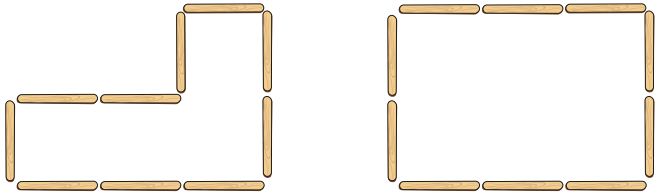
The area is 32 cm²

Do you agree with Tiny?

Explain your answer.

No

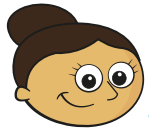
Dora has made two rectilinear shapes using lolly sticks.



The length of each lolly stick is 10 cm.

Work out the perimeter of each shape.

What do you notice?



If I cut a rectangle out of the corner of another rectangle, the perimeter of the rectilinear shape will always be the same as the perimeter of the rectangle I started with.

Do you agree with Dora?

Talk about it with a partner.

both 100 cm | Yes

Area of a triangle – counting squares

Notes and guidance

In this small step, children are introduced to finding the area of a triangle by counting squares. They estimated area in Year 5, but may need to be reminded of efficient strategies for calculating and estimating areas of shapes.

Children first find the areas of triangles that require them to only count full and half squares. They can calculate these separately and then combine them to find the area. They then move on to estimating the areas of triangles that involve sections of squares greater and less than half. Children also explore creating their own triangles with a specific area.

Some links are made between the area of a rectangle and the area of a triangle, but the formula is not introduced until the next step.

Things to look out for

- Children may count half squares as full squares.
- Without an efficient method, children may not count squares accurately.
- Children may find it difficult to draw a triangle with a specific area.
- If a triangle is not placed on a line, children may believe it is impossible to estimate its area.

Key questions

- How is finding the area of a triangle similar to finding the area of a rectangle when counting squares? How is it different?
- How will you count the squares accurately?
- Is more or less than half the square shaded?
- Can you see any parts of squares that combine to make approximately one full square?
- How does the area of the rectangle link to the area of a triangle? Why do you think this happens?

Possible sentence stems

- The triangle has _____ full squares.

  The triangle has _____ half squares.

  The area of the triangle is _____ cm^2
- The approximate area of the triangle is _____ cm^2

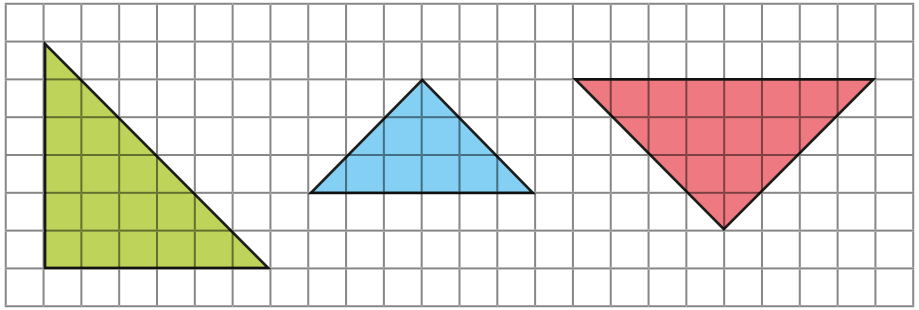
National Curriculum links

- Calculate the area of parallelograms and triangles

Area of a triangle – counting squares

Key learning

- Complete the sentences to find the area of the triangles.



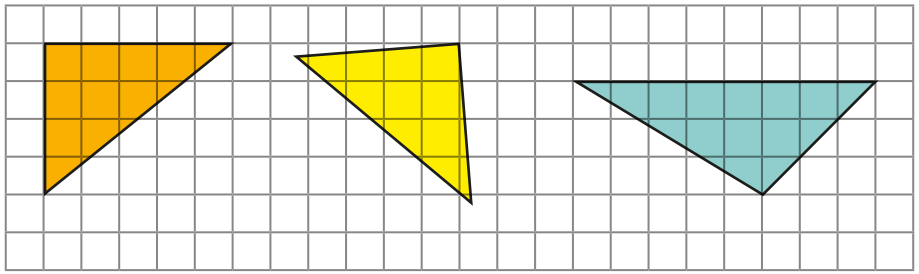
The triangle has _____ full squares.

The triangle has _____ half squares.

_____ + _____ = _____

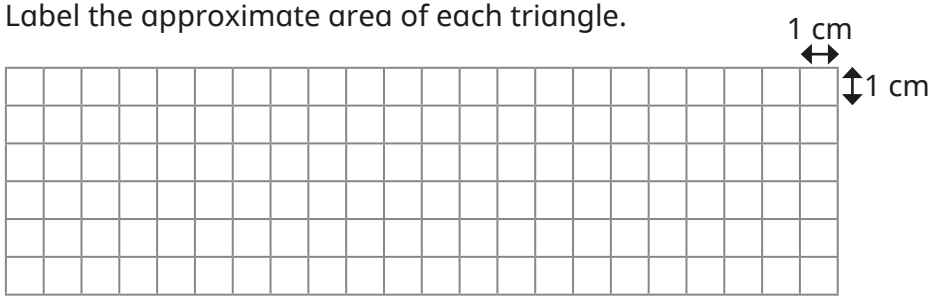
The total area of the triangle is _____ cm^2

- Estimate the areas of the triangles by counting squares.

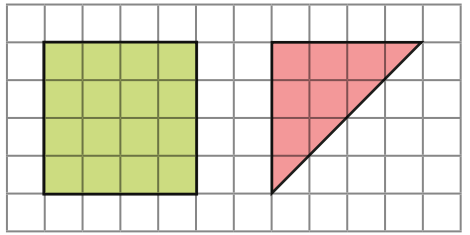


- Draw three different triangles that have an area between 5 cm^2 and 15 cm^2

Label the approximate area of each triangle.



- Work out the area of each shape by counting squares.



What do you notice about the area of the triangle compared to the area of the square?

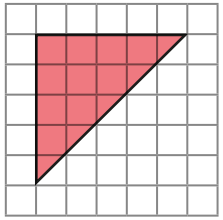
Does this always happen?

Draw a rectangle and a triangle to explore the pattern.

Area of a triangle – counting squares

Reasoning and problem solving

Tiny says that the area of the triangle is 15 cm^2

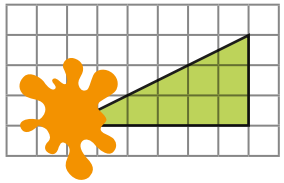


Tiny is incorrect.

Explain what Tiny has done wrong.

Tiny has counted the half squares as full squares.

Part of the triangle has been covered.

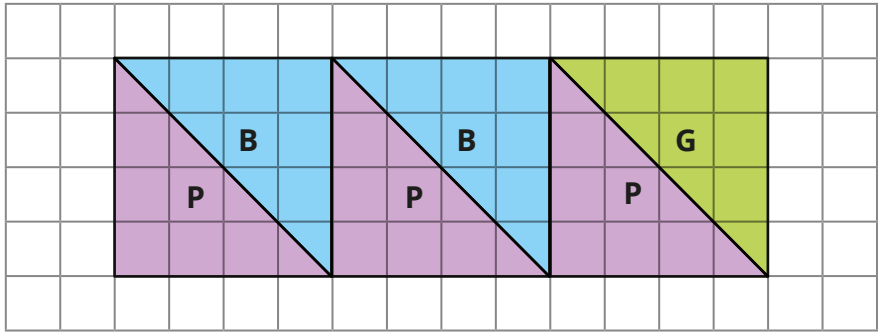


Estimate the area of the whole triangle.

Would your estimate change if the splat was in a different place?

9 cm^2

Huan draws three squares and splits them into six right-angled triangles.



What is the total area of the purple (P) triangles?

What is the total area of the blue (B) triangles?

What is the area of the green (G) triangle?

Compare methods with a partner.

purple: 24 cm^2

blue: 16 cm^2

green: 8 cm^2

Area of a right-angled triangle

Notes and guidance

In this small step, children look in more detail at finding the areas of right-angled triangles.

Children move on from counting squares to identifying and using a formula. They explore the fact that a right-angled triangle with the same length and perpendicular height as a rectangle has an area that is half the area of the rectangle. They then adapt the formula for the area of a rectangle to find the area of a right-angled triangle. Children use the formula area = $\frac{1}{2}$ × base × perpendicular height rather than $\frac{1}{2}$ × length × width in readiness for the next step, where they look at non-right-angled triangles. This vocabulary should be explored and children should be confident identifying the correct parts of the triangle.

Things to look out for

- Children may not identify that a rectangle can be made into two right-angled triangles.
- Children may not be able to identify the base and perpendicular height, choosing the incorrect measurements to multiply.
- Children may not associate multiplying by $\frac{1}{2}$ with dividing by 2

Key questions

- How can you split the rectangle into two right-angled triangles?
- What do you notice about the two triangles?
- What do you notice about finding the area of a rectangle and finding the area of a right-angled triangle?
- What is the formula to find the area of a right-angled triangle?
- What does “perpendicular” mean?
- How do you know which measurement is the base/ perpendicular height?

Possible sentence stems

- The area of the right-angled triangle is _____ the area of the rectangle.
- The formula for the area of a triangle is …

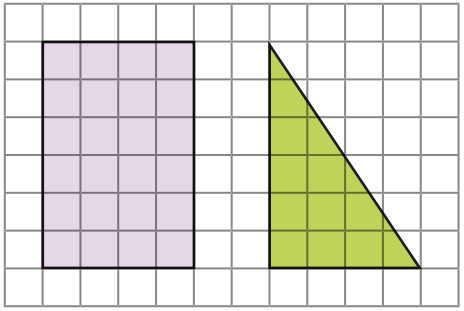
National Curriculum links

- Recognise when it is possible to use formulae for area and volume of shapes
- Calculate the area of parallelograms and triangles

Area of a right-angled triangle

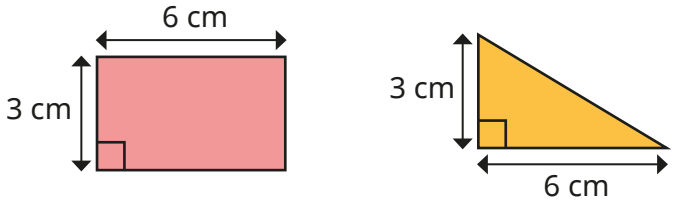
Key learning

- Here is a rectangle and a right-angled triangle.



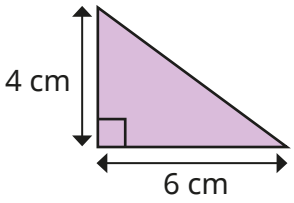
  - What is the area of the rectangle?
  - What is the area of the right-angled triangle?
  - What do you notice?

- Here is a rectangle and a triangle.



  - What is the area of the rectangle?
  - What is the area of the triangle?
  - How do you work out the area of a right-angled triangle?

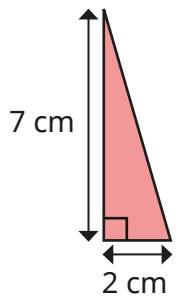
- Scott uses the formula to work out the area of this right-angled triangle.

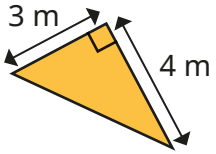


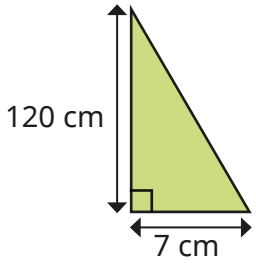
$$\text{area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

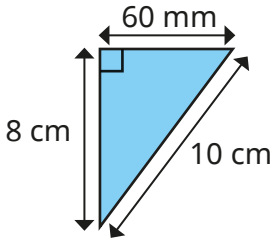
$$\text{area} = \frac{1}{2} \times 6 \times 4 = \frac{1}{2} \times 24 = 12 \text{ cm}^2$$

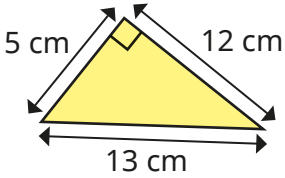
Use the formula to find the areas of the triangles.

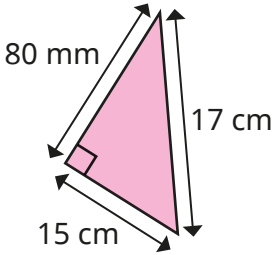








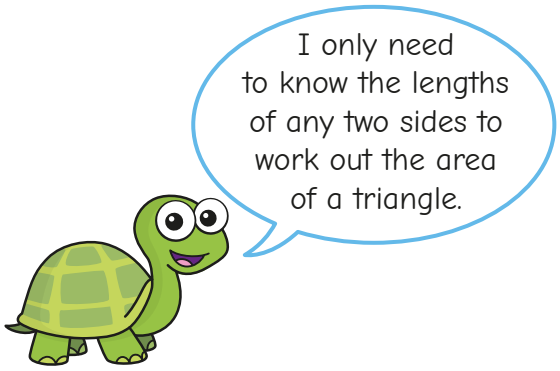




Area of a right-angled triangle

Reasoning and problem solving

Tiny is working out the area of a right-angled triangle.



Do you agree with Tiny?

Explain your answer.

No

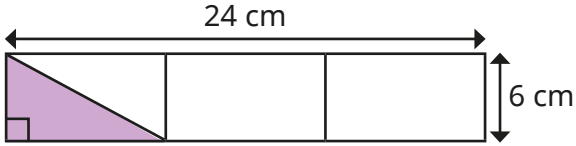
The area of a right-angled triangle is 54 cm^2

What could the base and height be?

How many solutions can you find?

multiple possible answers, e.g.
18 cm and 6 cm

Calculate the area of the shaded triangle.



Compare methods with a partner.

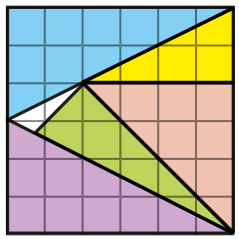
24 cm^2

Aisha has placed five right-angled triangles onto a square.

The total area of the square is 36 cm^2

1 cm^2 is not covered by a triangle.

What is the area of the green triangle?



5 cm^2

Area of any triangle

Notes and guidance

In this small step, children extend their knowledge of finding the area of a right-angled triangle to find the area of any triangle.

Children use the same formula as before, but now need to identify that the perpendicular height is not always the length of one of the sides. Initially, they find the areas of triangles where only the base and perpendicular height are given, before looking at triangles where more measurements are given.

Children need to understand that the base is not always at the bottom of a triangle and sometimes there may be more than one possible calculation they could use to find the area.

Things to look out for

- Children may not identify the base and perpendicular height correctly.
- Children may think that the base is always at the bottom of the triangle.
- Children may think that the measurement giving the perpendicular height is always labelled inside the triangle.
- If given more than two measurements, children may multiply the incorrect lengths.

Key questions

- What is the formula for the area of a triangle?
- How do you know which side is the base?
- How do you know what the perpendicular height is?
- How do you know that you are using the correct lengths?
- Is there more than one way to find the area of this triangle?
- Is the base always at the bottom of the triangle?

Possible sentence stems

- The formula for the area of a triangle is ...
- The base is _____ cm.

  The perpendicular height is _____ cm.

  Area = $\frac{\square}{\square}$ × _____ × _____

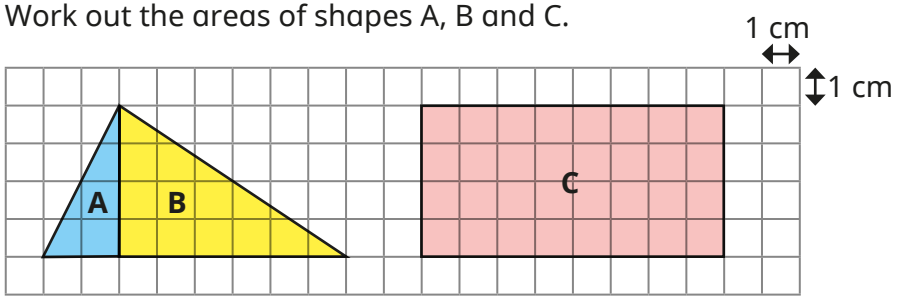
National Curriculum links

- Recognise when it is possible to use formulae for area and volume of shapes
- Calculate the area of parallelograms and triangles

Area of any triangle

Key learning

- Work out the areas of shapes A, B and C.

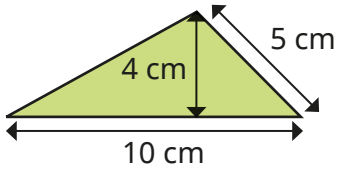


  - What is the total area of the scalene triangle formed by A and B?
  - Compare this area to the area of rectangle C.

    What do you notice?

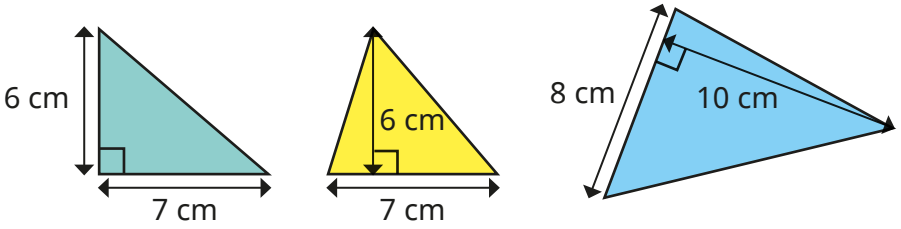
    Does this always happen?

- Here is a triangle.



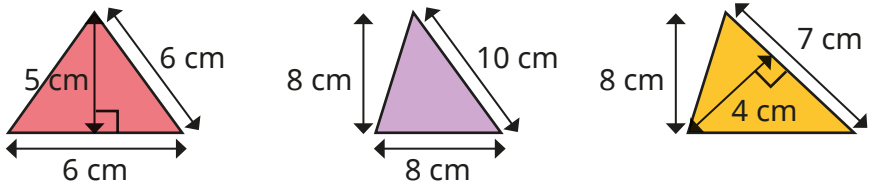
  - What is the length of the base of the triangle?
  - What is the perpendicular height of the triangle?
  - Use the formula area = $\frac{1}{2}$ × base × perpendicular height to work out the area of the triangle.

- Work out the areas of the triangles.

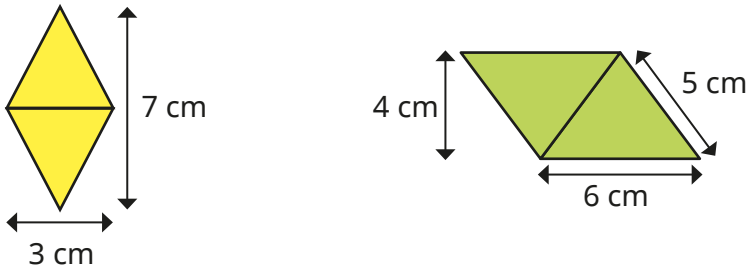


  What is the same and what is different about the first two triangles?

- Find the area of each triangle.



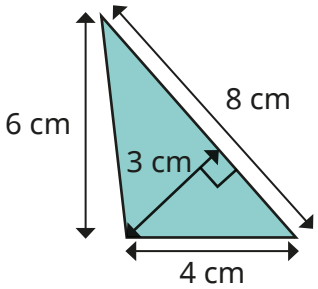
- Calculate the area of each shape.



Area of any triangle

Reasoning and problem solving

Tiny is finding the area of this triangle.



I need to multiply all the lengths, then divide by 2

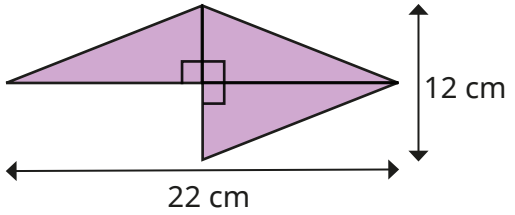
Explain why Tiny is incorrect.

Work out the area of the triangle.

Can you find more than one way to do it?

$12\ cm^2$

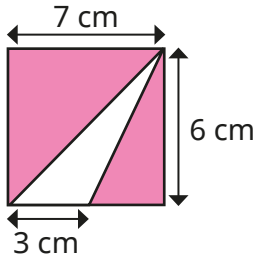
This shape is made up of three identical triangles.



What is the area of the shape?

$99\ cm^2$

Here is a flag.



Find the area of the flag that is white.

Is there more than one way to find the answer?

$9\ cm^2$

Area of a parallelogram

Notes and guidance

In this small step, children explore the area of a parallelogram, identifying and using a formula.

Children look at the properties of a parallelogram and compare to a rectangle. Using the "cut-and-move method", they explore how the parts of the parallelogram can be rearranged to make a rectangle in which the length and width correspond to the base and perpendicular height of the parallelogram. Through this, they recognise that the area of a parallelogram can be found by using the formula area = base × perpendicular height.

As they did for triangles, children need to be able to identify the base and perpendicular height when given more than the required measurements. This needs to be carefully modelled so that children do not believe that area = $l \times w$. It may be useful to compare all the formulas they know for finding the areas of shapes.

Things to look out for

- When finding the area of a parallelogram, children may try to use the formula for finding the area of a rectangle or a triangle.
- Children may struggle to identify the base and perpendicular height.

Key questions

- How could you change the parallelogram into a rectangle? How will this help you to find the area?
- How can you count the squares accurately to find the area?
- How do you know you have found the base/ perpendicular height?
- What is the formula for finding the area of a parallelogram?
- When you have different units, what is your first step?

Possible sentence stems

- The base of the parallelogram is _____ cm.

  The perpendicular height of the parallelogram is _____ cm.

  The area of the parallelogram is _____ × _____ = _____ cm^2

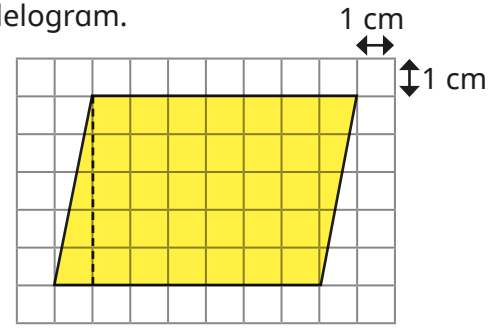
National Curriculum links

- Recognise when it is possible to use formulae for area and volume of shapes
- Calculate the area of parallelograms and triangles

Area of a parallelogram

Key learning

- Here is a parallelogram.



  - Copy the parallelogram onto centimetre squared paper.

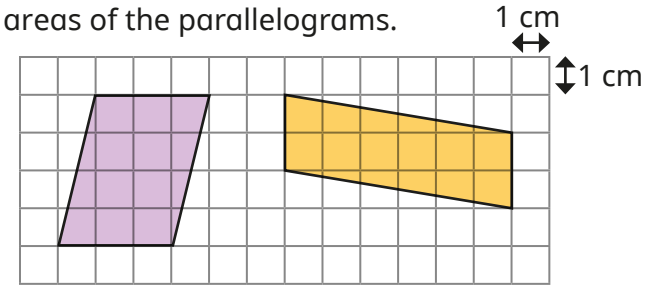
    Estimate its area by counting squares.
  - Now cut along the dotted line.

    Move the triangle to make a rectangle.

    What is the area of the rectangle?

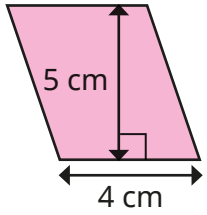
    What do you notice?

- Work out the areas of the parallelograms.



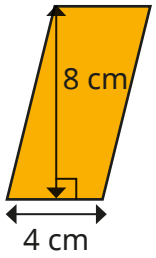
Explain your method to a partner.

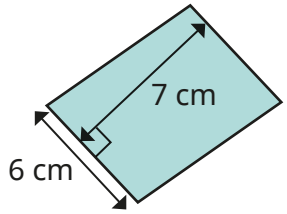
- Annie has worked out the area of this parallelogram.

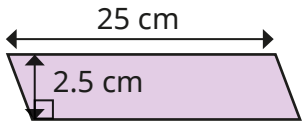


area = base × perpendicular height
= 4 cm × 5 cm
= 20 cm^2

Use Annie's method to find the areas of the parallelograms.

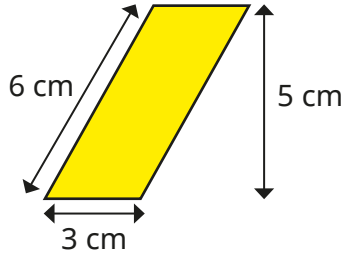


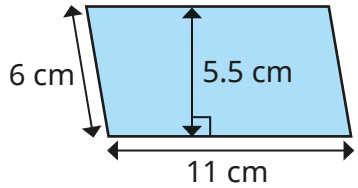




- Label the base b and perpendicular height h on each parallelogram.

Then find the area of each shape.



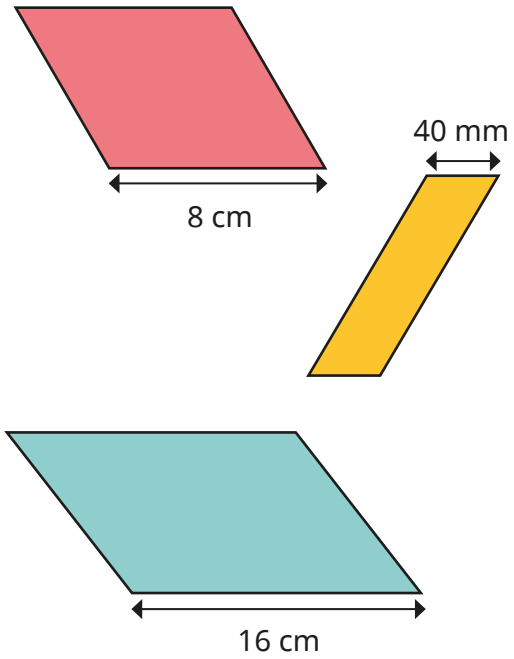


Area of a parallelogram

Reasoning and problem solving

These parallelograms each have an area of 40 cm^2

Find the perpendicular height of each shape.



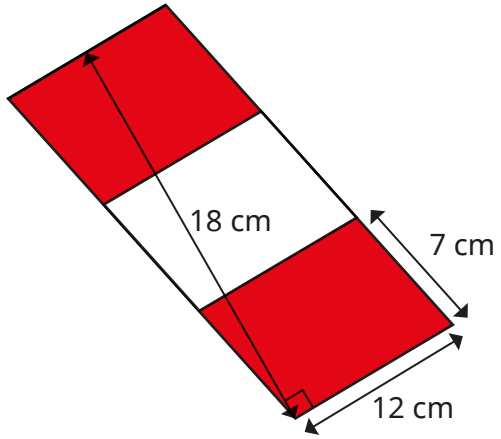
5 cm

10 cm

2.5 cm

All the parallelograms have the same area.

Find the total area of the shaded parallelograms.



144 cm^2

7 cm

Which measurement is not needed?

Find more than one method to work out the answer.

Which was more efficient?

Volume – counting cubes

Notes and guidance

In Year 5, children began to explore volume as the amount of space that a solid object takes up. They started by counting cubes, before being introduced to cubic centimetres (cm^3) as a unit of measure for volume. This learning is recapped at the beginning of this small step.

Children then explore shapes where they can find the volume by multiplying the volume of a single layer by the number of equal layers. This can include cuboids and other prisms. Encourage children to explore the relationship between the total volume of a cuboid and its length, width and height, although there is no need to explicitly introduce the formula for finding the volume of a cuboid, as this will be covered in more detail in the next step.

Things to look out for

- Children may believe that shapes that look different visually must have different volumes.
- Children may ignore cubes that cannot be "seen" in an image, so it is important to discuss the possibility of hidden cubes and how children might know for certain that more cubes exist even if they cannot see them.

Key questions

- What is volume?
- How is volume different from area?
- How can you count the number of cubes efficiently?
- If each cube has a volume of 1 cubic centimetre (cm^3), what is the volume of the shape?
- How many cubes are there in this layer? How many equal layers are there? So how can you find the volume?
- What is the length/width/depth of this cuboid?

Possible sentence stems

- The volume of the shape is _____ cubes.
- The volume of the shape is _____ cm^3
- There are _____ cubes in each layer and _____ equal layers, so the volume is _____ cubes.

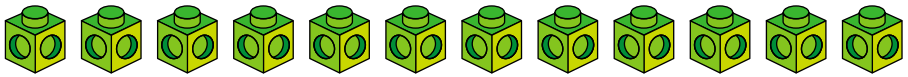
National Curriculum links

- Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units

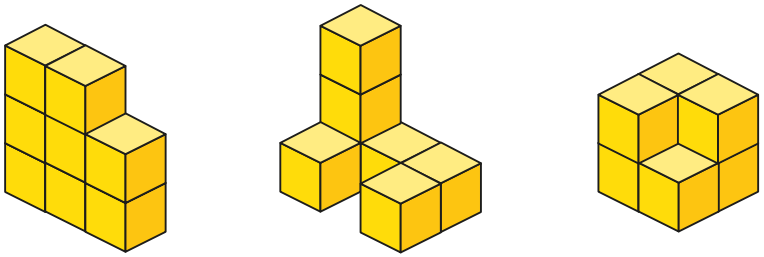
Volume – counting cubes

Key learning

- Using 12 cubes, how many different shapes can you make?

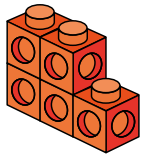


- How many cubes are used to make each shape?



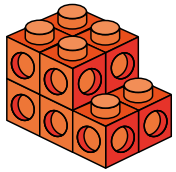
- Brett makes this shape using cubes.

  What is the volume of the shape in cubes?



  Mo makes an identical shape and attaches the shapes together like this.

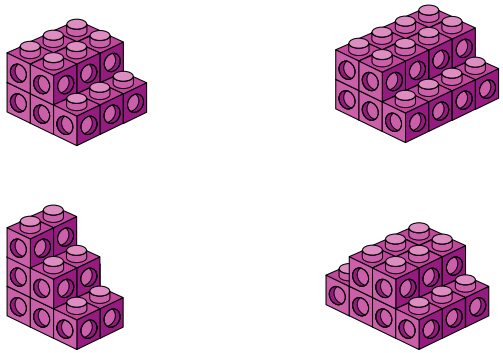
  What is the volume of the shape in cubes?



  What do you notice?

- Each shape is made using centimetre cubes.

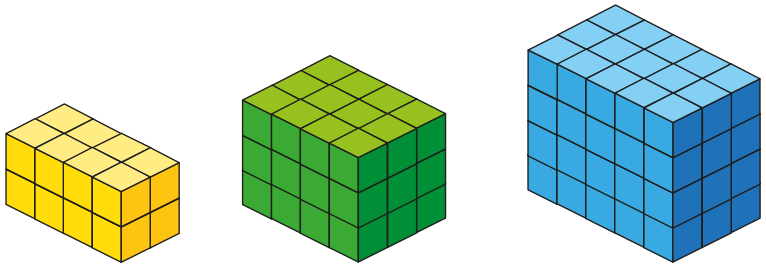
  Work out the volume of each shape in cm^3



  What is the quickest way of finding the volumes?

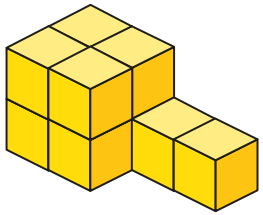
- Each cuboid is made using centimetre cubes.

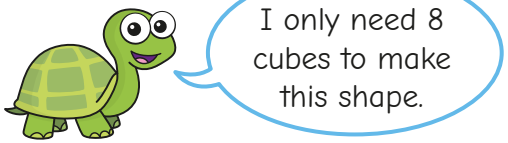
  Find the volumes of the cuboids.



Volume – counting cubes

Reasoning and problem solving





Do you agree with Tiny?

Explain your reasons.

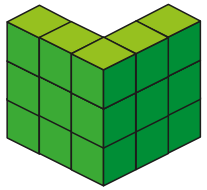
No

Make a cuboid using 24 cubes.

What are the dimensions of your cuboid?

How many different cuboids can you make with this number of cubes?

multiple possible answers, e.g.
6 cubes, 2 cubes and 2 cubes

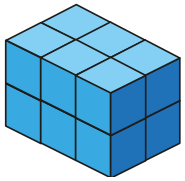


What could the volume of this shape be?

Compare answers with a partner.

between 15 and 23 cubes

Dani makes this cuboid.



She makes another cuboid by increasing the height, width and depth by 1 cube.

What is the difference in the volumes of the cuboids?

24 cubes

Volume of a cuboid

Notes and guidance

In this small step, children move on from counting cubes to finding the volumes of cuboids using multiplication and applying a formula.

Children discover that they can use multiplication to find the number of cubes in one "layer" of the shape and then multiply this by the number of layers to find the total volume. This will help children identify the formula: volume of cuboid = length × width × height. They should recognise that the formula works whichever way they look at the cuboid and what they think of as a "layer".

Once children understand the formula, encourage them to find the most efficient method to calculate the volume using the associative law of multiplication.

Things to look out for

- Children may think that it is impossible to find the volume without cubes.
- Children may think that they must always multiply $l \times w \times h$ in that order, which may not always be the most efficient calculation.
- When finding the volumes of cubes, children may think that they need more than one measurement.

Key questions

- What is volume?
- How many cubes are there in one layer? How do you know?
- How do you find the total volume of the cuboid?
- What is the formula to find the volume of a cuboid?
- What is the same and what is different about area and volume?
- What is the most efficient order to multiply the three numbers together?

Possible sentence stems

- There are _____ cubes in each layer.

  There are _____ layers.

  The volume of the cuboid is _____

- The length is _____. The width is _____. The height is _____.

  The volume of the cuboid is _____ × _____ × _____ = _____

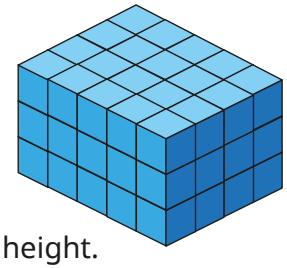
National Curriculum links

- Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units

Volume of a cuboid

Key learning

- The cuboid is made using centimetre cubes.



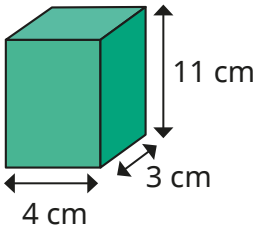
  - What is the volume of the cuboid?
  - What is the length, width and height of the cuboid?
  - Find the product of the length, width and height.

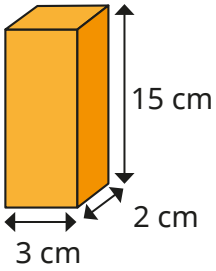
  What do you notice?

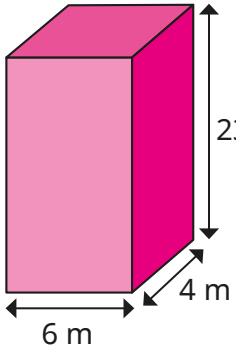
- Here is the formula for the volume of a cuboid.

  volume = length × width × height

  Use the formula to find the volumes of the cuboids.

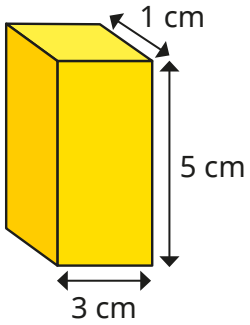


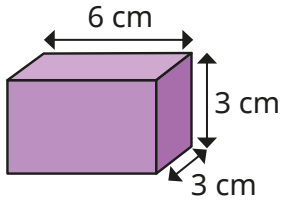


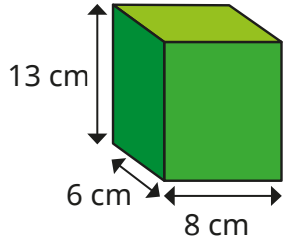


  Does it matter in which order you multiply the numbers?

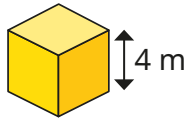
- Find the volumes of the cuboids.

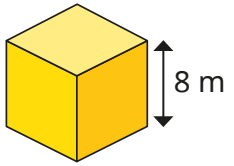






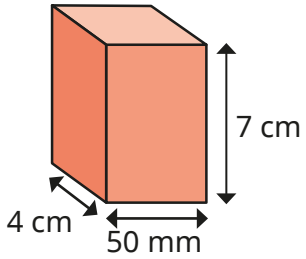
- Find the volumes of the cubes.

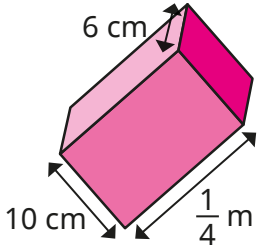


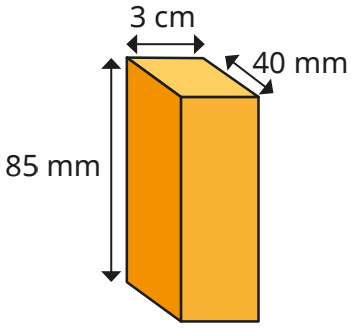


  Compare methods with a partner.

- Find the volumes of the cuboids.



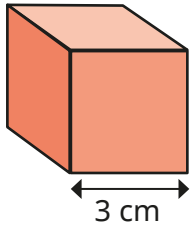


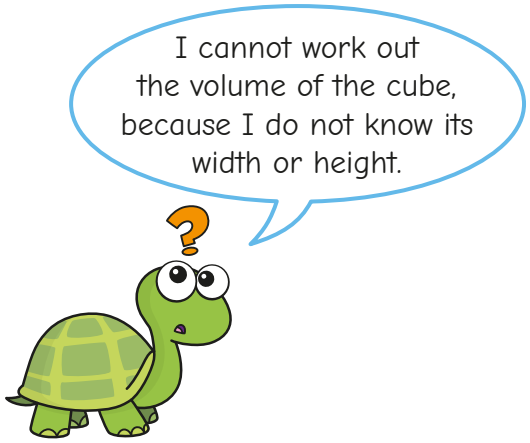


Volume of a cuboid

Reasoning and problem solving

Here is a cube.



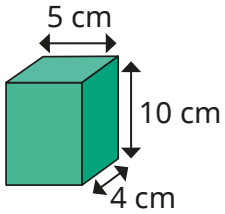


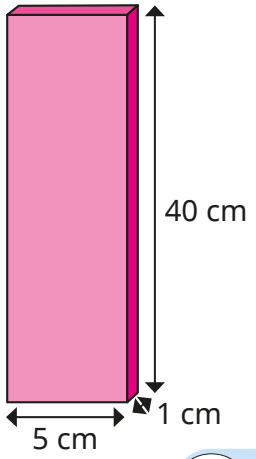
Do you agree with Tiny?

Explain your answer.

No

Which cuboid has the greater volume?

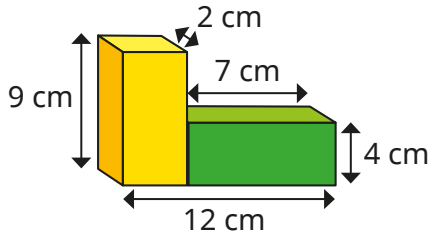




Explain how you know.

Both cuboids have the same volume: 200 cm^3

Calculate the volume of the compound shape.



146 cm^3